

## **Abstract Experimental Models For Analyzing Mobility-Fertility Data**

The data reproduced in Table 1, first reported by Berent (1951), have recently been reanalyzed by Duncan (1956) and Hope (1971). By replacing these data by different abstract experimental models, these authors have reached conclusions that are not in agreement with each other. The purpose of this paper is to examine the validity of the models employed in their analyses by these authors, and to suggest possible alternatives that deserve to be considered when analyzing data such as these.

A brief description of the data in Table I may be in order. These are from the Social Survey in England and Wales in 1949 in which a nation-wide sample of approximately 10,000 cases was studied. Table 1 is confined, however, to couples who had been married 20 years or longer by the time of the survey.

The top half of Table 1 contains the average number of live births (referred to below as fertility or family size) per couple in each origin-destination category. Origin here refers to the social (=occupational) class of the husband's father, and destination, the social (=occupational) class of the husband at the time of the survey. The bottom half of the table shows the number of couples in the sample in each origin-destination category. It is relevant to note that since all couples included in Table 1 had been married 20 years or more, it is valid to consider that their family growth had been virtually completed by the time of the survey.

### **Berent-Duncan-Hope Model**

A general pattern that is clearly discernible in the top half of Table 1 is that the figures increase in each row from left to right (up-mobiles, non-mobiles, and down-mobiles, in that order), while in each column the figures increase from top to bottom (down-mobiles, non-mobiles, and up-mobiles in that order). Berent (1951) attributed this pattern to "the acquisition of the fertility characteristics of the class into which the sons have moved and the maintenance by them of the family building habits of the class in which they were born", (p.248) The idea behind this interpretation appears at first sight quite simple. Each social class is thought of as having associated with it certain family building habits. It is assumed that the fertility of members of any given social class will be influenced by the family building habits characteristic of that class. It is also assumed that the effect of exposure to the

family building habits of any given class would continue to be in operation, at least in part, even after mobility occurs. (The need for this second assumption will become clear later on.) Thus each couple is believed to be subjected to two influences (effects), namely, those of the family building habits characteristic of the classes of origin and destination respectively. This reasoning leads to the following statistical model:

$$E(y_{ij}) = g + \alpha_i + \beta_j$$

where

- $E(y_{ij})$  = the expected value of the average fertility (number of live births) in the (i, j) cell (i th class of origin and j th class of destination), meaning by expected value the value one would expect had there been no sampling fluctuation,
- $g$  = the effect of all factors that uniformly affect all couples, irrespective of the classes of origin and destination,
- $\alpha_i$  = the effect (increment or decrement to family size) attributable to the family building habits characteristic of the i th class of origin, and
- $\beta_j$  = the effect (increment or decrement to family size) attributable to the family building habits characteristic of the j th class of destination.

Notice that according to Model (1), the average family size in each column of Table 1 should increase consistently from top to bottom if  $\alpha$ 's consistently increase from class of origin 1 (High) through class of origin 4 (Low). Similarly, Model (1) implies that the average family size in each row should increase from left to right if  $\beta$ 's increase consistently from destination class 1 (High) through destination class 4 (Low). So Berent's interpretation of the patterns in Table 1 in terms of the acquisition by the sons of the fertility habits of their class of destination and the maintenance by them of the fertility habits of their class of origin can be validated if we can show

- (1) that Model (1) is an adequate representation of the patterns in Table 1,
- (2) that  $\alpha$ 's increase in the order  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ ,
- (3) that  $\beta$ 's increase in the order  $\beta_1, \beta_2, \beta_3, \beta_4$ .

When Model (1) was fitted to the data in Table 1, the following estimates of relative values of  $\alpha$ 's and  $\beta$ 's were obtained (see Hope, 1971:1022; Duncan, 1966:96):

Estimate of $(\alpha_2 - \alpha_1)$	=	0.37
Estimate of $(\alpha_3 - \alpha_1)$	=	0.57
Estimate of $(\alpha_4 - \alpha_1)$	=	1.00
Estimate of $(\beta_2 - \beta_1)$	=	0.10
Estimate of $(\beta_3 - \beta_1)$	=	0.67
Estimate of $(\beta_4 - \beta_1)$	=	1.26

These estimates validate Berent's interpretation, *provided that the fit of Model (1) to the data in Table 1 can be regarded as adequate*. As regards the goodness of fit of Model (1) to the data in Table 1 there is some difference of opinion. Duncan (1966) believes that the fit is adequate, but Hope (1971) disagrees. This disagreement will be discussed later herein.

It deserves to be emphasized that Berent (1951) does not explicitly make use of Model (1) to arrive at his interpretation described above of the patterns in Table 1, but there is no doubt that he considers Model (1) as an appropriate representation of the data in Table I. In the appendix to his report he actually fits this model to the data and presents the results.

Duncan (1966) makes use of Model (1) for the specific purpose of shedding light on the mobility-fertility hypothesis. Interpreting mobility-fertility hypothesis as one that declares the existence of "mobility effect on fertility", Duncan suggests that if Model (1) adequately represents the data in Table 1, the mobility-fertility hypothesis should be rejected. "One is not entitled to discuss 'effects' of mobility (or other status discrepancy measures) until he has established that the apparent effect cannot be due merely to a simple combination of effects of the variables used to define mobility." (Duncan, 1966:91) In Duncan's view, the mobility-fertility hypothesis is thus equivalent to the statistical hypothesis which holds that the effects of the class of origin and the class of destination on fertility are not additive. (See also Blau and Duncan, 1967, Ch. 11). In this respect Hope (1971) agrees with Duncan (1966). Hope's quarrel with Duncan has to do with the question whether the  $\alpha$ 's in Model (1) would be distinct from the  $\beta$ 's. Hope advocates that each class be assigned only one parameter, say  $C_1$ , whether it is the class of origin or of destination. According to this view Model (1) should be replaced by

$$E(y_{ij}) = g + c_i + c_j \quad (2)$$

where  $C_i$  ( $C_j$ ) represents the effect (increment or decrement to family size) attributed to social (=occupational) class  $i$  ( $j$ ). The mobility-fertility hypothesis is then considered as equivalent to the existence of systematic, *i.e.*, identifiable in terms of mathematical functions of  $c_i$  and  $C_j$ , departures of the observed means in Table 1 from the corresponding values implied by Model (2). Hope makes a persuasive case for the existence of systematic departures of the kind he believes represent "mobility effects". Duncan, on the other hand, did not believe at the time he wrote his 1966 paper that there are systematic departures from what Model (1) "predicts". There is no disagreement between Duncan and Hope on the point that it is the existence of non-additivity in the patterns in Table 1 that one should look for.

As far as statistical principles are concerned, I can see therefore only one major difference between the views of Duncan and Hope, and that concerns whether  $\alpha$ 's should be distinct from  $\beta$ 's in Model (1). It may not be out of place to discuss this issue briefly. I shall argue that they should be distinguished.

To argue my case, let me start by emphasizing that when an analyst talks about the appropriateness of a model to represent a given set of data, he is implicitly talking about the validity of an abstract experimental model as the replacement of the concrete data set. It would be instructive in this connection to think of the data in Table 1 as the result of a two-factor experiment, Factor A standing for the class of origin and Factor B for the class of destination. Each factor has four levels (categories). The advantage of viewing the data in Table 1 as the result of a two-factor experiment is that it would help retain in view only those features of the case which we propose to deal with by statistical methods.

A question that immediately comes to mind regarding this experimental model is whether we are to suppose that the couples in the sample have been exposed to factors A and B simultaneously or sequentially (*e.g.* B after A). My own view is that it is more valid to suppose that each person in Berent's sample was exposed to Factor A first and later on to factor B, rather than to both of them simultaneously. It is logical to regard the class-linked family-building habits of the class of origin as those of the older generation, whereas the corresponding habits of the class of destination are to be regarded as those characteristic of the current generation. A person is exposed to the class-linked family-building habits of his/her

father's generation while he/she is growing up, his/her exposure to the class-linked family-building habits of his/her own generation occurs later. If, then Factor A and Factor B are to be regarded as having been applied sequentially, we need to distinguish between what statisticians call "carryover" and "direct" effects of treatments. The meaning of "carryover" effect is similar to what we mean by the term "hang-over" from last night's drinking. It is the effect that lingers beyond the period of application of treatment. "Direct" effect is the effect of a treatment that operates during the period in which the treatment is applied. (See Namboodiri, 1972, for a review of relevant literature on the subject.) It is not logical to assume that the "carry-over" effect is necessarily equal to the "direct" effect of any treatment. Hence, *even if it is valid to assume that there has been no change in the class-linked family-building habits from one generation to the next*, it is not valid to assume that the "carry-over" effect of the class-linked family-building habits of the older generation will be equal to the "direct" effect of the class-linked family-building habits of the current generation until evidence to the contrary becomes available. Now when it is realized that Model (1) is equivalent to the equation

$$E(y_{ij}) = g + (\alpha_i = \text{"carry-over" effect of class of origin } i) + (\beta_j = \text{"direct" effect of class of destination } j) \quad (3)$$

It can be easily appreciated in the light of the above discussion why one needs to distinguish between  $\alpha$ 's on the one hand and  $\beta$ 's on the other.

For the present purpose there is no need to continue to use the terms "carry-over" effects and "direct" effects. Therefore,  $\alpha$ 's and  $\beta$ 's will be referred to below simply as the effects of classes of origin and destination respectively.

What remains to be examined is whether it is valid to equate the mobility-fertility hypothesis with the hypothesis of non-additivity between the effects of the classes of origin and destination. I shall argue that it is not valid to do so.

As mentioned already, the null hypothesis is that non-additivity between the effects of the classes of origin and destination is absent is equivalent to the hypothesis that Model (1) is an adequate representation of the data in Table 1. That Model (1) is an adequate representation of the patterns in Table 1 implies and is implied by, except for the direction of causality, the following hypotheses:

- (1) Fertility comparisons between two or more classes of destination in any given class of origin are identical to the corresponding comparisons in any other class of origin. For example, the difference between average family sizes of classes of destination 1 and 4 will be identical in all four classes of origin, since this should be equal to the difference between  $\beta$  and  $\beta_4$  in all classes of origin.
- (2) Fertility comparisons between two or more classes of origin in any given classes of destination are identical to the corresponding comparisons in any other class of destination. For example, the difference between average family sizes of classes of origin 1 and 2 will be identical in all four classes of destination, since this should be equal to the difference between  $\alpha_1$  and  $\alpha_2$  in all classes of destination.

These two hypotheses are respectively equivalent to the following:

- (1') The relationship between B (class of destination) and C (fertility) remains identical in all categories of A (class of origin).
- (2') The relationship between A (class of origin) and C (fertility) remains identical in all categories of B (class of destination).

Now suppose that Model (1) has been found to be a satisfactory representation of the patterns in Table 1. It means that the patterns in Table 1 are consistent with both Hypothesis (P) and Hypothesis (2') described above. But does this mean that the data are not consistent with the mobility-fertility hypothesis? The answer is affirmative if the mobility-fertility hypothesis represents the opposites of Hypotheses (P) and (2'). In my judgment this is not the case. The main thrust of the mobility-fertility hypothesis is that the chance of up-mobility will be relatively large if family size is small and relatively small if family size is large. Thus, according to the classical formulation of Dumont, "just as a column of liquid has to be thin in order to rise under the force of capillarity, so a family must be small in order to rise in the social scale", (quoted by Westoff, 1953.) Or, according to the description given in the United Nations' *The Determinants and Consequences of Population Trends*, "social mobility is...more feasible with one or two children than with a larger number". (1253:79) Or, according to Westoff, *et al.*, "the general hypothesis [is] that upward mobility is associated with lower fertility". (1963:134) What I read in these quotes is the following description of the mobility-fertility hypothesis: The probabilities of transition from a given class of origin to various classes of destination are dependent on the number of children in the family the probability of moving up to any given class varying negatively with family size.

When put this way it appears that the hypothesis regards mobility (the transition probabilities referred to above) as the dependent variable, while the class of origin and fertility are regarded as independent variables. The Duncan-Hope formulation of the hypothesis, on the other hand, views fertility as the dependent variable while the classes of origin and destination are regarded as independent variables.

If it is valid to assume that family growth is completed before mobility is accomplished, if at all, then the first model is the appropriate one, whereas if mobility precedes family formation and/or growth, the appropriate model is the second one. But when mobility is determined as was done in Berent's study by comparing the husband's father's occupation with the husband's occupation 20 or more years after marriage, it does not seem to be valid to assume sequentiality between mobility and family growth; a more valid assumption is one of simultaneity between the occurrence of mobility and family growth. If we regard Berent's data as resulting from a process in which mobility occurred at the same time as family was growing, then the relevant abstract experimental design applicable to the analysis of the data is that of a single-stimulus-double-response experiment, the class of origin being regarded as the stimulus and the class of destination and family growth as symmetrically related responses. See Figs, 1a and 1b.

#### **Berent's Data Viewed As Generated By Single-Stimulus-Double-Response Experiment**

When we try to apply this model to Berent's data, two problems raise themselves. One is that the response variables (mobility and fertility) are not measured on interval scale. Even if we are willing to regard the number of live births as interval scale data, we are still faced with the problem that the mobility data are in the form of frequency distributions. As a solution to this problem I suggest that we treat both the response variables as categorical (qualitative) variables. That is, the fertility data are to be viewed as frequency distributions by family size, and the mobility data are to be viewed as frequency distributions by class of destination. We can then apply to the data the techniques that are applicable for the multi-variate analysis of qualitative data.

A second problem that we face in Berent's data is that we do not have full data on fertility of the couples in the origin-destination cells in Table 1. We are given only the mean family size of couples in each cell. What we need is a set of frequency distributions as exemplified in Table 2. The particular method by which the frequencies in Table 2 were arrived at is not relevant here. What is pertinent to note is that the data in Table 2 are consistent with Berent's (1951) Table 1 (which is the same as Table 1 herein), and are also consistent with the information in Berent's (1951) Table 4, the relevant features of which are reproduced in Table 3. There is, however, no guarantee that Table 2 is a true reproduction of Berent's raw data. But our illustrative purpose will be adequately served by Table 2. The results of fitting different models to the data in Table 2 are reported below.

First let us analyze the data in Table 2 in the manner suggested by Duncan (1966), and Blau and Duncan (1967). This amounts to fitting Model (1). The results are presented in Table 4.

Note that the total sum of squares (9456.7) in Table 4 falls short of the corresponding figure (10,074.3) reported by Berent (1951). The cell frequencies in Table 2 could be readjusted so as to make the total sum of squares equal to 10,074.3. But that was not felt worthwhile in view of the fact that any such adjustments would be extremely arbitrary.

Since the non-additivity sum of squares in Table 4 is not significant, we should declare that the mobility-fertility hypothesis, as interpreted by Duncan (1966), and Blau and Duncan (1967), is not supported by the data in Table 2.

In fitting Model (1) to the data in Table 2, we have regarded the numbers of children as data on interval scale. We shall now regard the same data as multinomial. Since the frequencies in some of the cells in Table 2 are extremely small we shall condense the family-size categories into three: small (0-1), medium (2-3), and large (4 or more). When Table 2 is condensed in this way we get the three-way classification presented in Table 5.

We shall examine the fit of the data in Table 5 to two models. One corresponds to the hypothesis that non-additivity is absent in Table 1. The other states that the class of destination and family size are probabilistically independent in each class of origin. This latter hypothesis, it may be recalled is the null hypothesis that, except for the causal implication, corresponds to the substantive hypothesis implied in such statements as "a family must be small in order to rise in the social scale" (Dumont, 1890).

Let us first consider the hypothesis that non-additivity is absent in Table 1. We have already seen that this hypothesis, as applied to Table 2 and hence to Table 5, is equivalent to the hypothesis that the association between B (class of destination) and C (family size) is of equal strength and direction in each category of A (class of origin). This is the same as saying that the ABC interaction is zero, if we view A, B, and C as qualitative variables. (See Goodman, 1970.)

To test the hypothesis that the ABC interaction is zero in Table 5, we first estimate the expected frequencies in each cell of Table 5 under that hypothesis. The maximum likelihood estimates (under the hypothesis that the ABC interaction is non-existent) of the expected frequencies in the cells of Table 5 can be obtained by the method known as "fitting the marginals." We find the expected frequencies that fit the observed AB, AC, and BC

marginals reproduced in Table 6. An iterative method is available for the purpose (see, for example, Bishop, 1969, or Goodman, 1970). The method can be described as follows:

Let  $E_{ijk}$  and  $O_{ijk}$  stand for the expected and observed frequencies in the  $(i, j, k)$  cell, i.e.,  $i$  th class of A,  $j$  th class of B, and  $k$  th class of C, in Table 5.  $E_{111}$  thus represents the expected frequency corresponding to the observed frequency  $O_{111}=27$ , in the  $(A=1, B=1, C=1)$  cell.  $E_{443}$  represents the expected frequency corresponding to the observed frequency  $O_{443}=108$ , in the  $(A=4, B=4, C=3)$  cell, and so on.

The observed AB marginals shown in Table 6 can be represented by the symbols  $O_{i.}$ , the suffix  $.$  indicating summation for the suffix it replaces,  $k$  in this case. Thus

$$O_{11.} = \sum_{k=1}^3 O_{11k} = 27 + 36 + 2 = 65,$$

$$O_{44.} = \sum_{k=1}^3 O_{44k} = 42 + 70 + 108 = 220,$$

and so on. Similarly the observed AC marginals shown in Table 6 can be represented by the symbols  $O_{i.k}$ , and those in the BC marginals by  $O_{.jk}$ .

Now to estimate  $E_{ijk}$  we first start with a set of initial estimates obtained by evenly distributing  $O_{ij.}$  among the categories of variable C. Thus the initial estimate of  $E_{111}$ ,  $E_{112}$ ,  $E_{113}$  are all taken as one-third of  $(O_{11.}=65)$ . From the initial estimates thus obtained we calculate the AC marginals. These marginals may not coincide with the observed AC marginals shown in Table 6. The initial estimates of  $E_{ijk}$  are then replaced by a new set of values as shown below.

$$E_{ijk} (2) = O_{i.} [ E_{ijk} (1) / E_{i.k} (1) ]$$

where  $E_{ijk} (2)$  are the replacement of the initial estimates  $E_{ijk} (1)$ ,  $E_{i.k} (1)$  are the AC marginals calculated from the initial estimates, and  $O_{i.k}$  are the observed AC marginals in Table 6. Note that this formula calls for distributing each AC marginal  $O_{j.u}$  to the B categories in proportion to the corresponding distribution implied in the estimates  $E_{ijk} (1)$

From the new set of estimates we calculate BC marginals and compare them with the observed BC marginals in Table 6. If they are not sufficiently close enough, a third set of estimates are obtained according to the following formula:

$$E_{ijk} (3) = O_{.jk} [ E_{ijk} (2) / E_{.jk} (2) ]$$

From the new estimates, AB marginals are calculated and compared with the observed AB marginals. If they are not sufficiently close together, another set of estimates are calculated according to the formula

$$E_{ijk} (4) = O_{ij.} [ E_{ijk} (3) / E_{ij.} (3) ]$$

The process is repeated until the estimated  $E_{ijk}$  values yield AB, AC, and BC marginals that are as close as desired to the corresponding observed marginals. Let us denote these by the symbol  $E_{ijk}$ . Once these estimates are at hand, the statistic to test the hypothesis that the ABC interaction is non-existent is calculated in the manner we calculate goodness-of-fit chi-squares.

$$X^2 = \sum_{ijk} \left( \frac{(E_{ijk} - O_{ijk})^2}{E_{ijk}} \right)$$

The value of this statistic is to be compared with the tabulated value of chi-squared distri-

bution corresponding to  $(I-1)(J-1)(K-1)$  degrees of freedom, where I, J, and K are the numbers of categories for A, B, and C variables respectively. In the present case  $I=4$ ,  $J=4$ , and  $K=3$ .

The estimated expected frequencies calculated in the above manner are shown in Table 7. The chi-squared value was found to be 21.12 which has a probability level of approximately .25. Hence the data in Table 5 are consistent with the hypothesis that the ABC interaction is non-existent, which is what we expected since we already saw that Model (1) was an adequate fit to the data in Table 2 (see Table 4).

Let us now test the hypothesis that the class of destination and family size are probabilistically independent in each class of origin. It can be shown that this hypothesis is equivalent to the joint hypothesis that the ABC interaction and BC interaction are nonexistent (see Goodman, 1970). This is also equivalent to stating that the frequency distribution in Table 5 is completely determined by the AB and AC marginals. The maximum likelihood estimates of the expected frequencies under this hypothesis are given by the following formula:

$$E_{ijk} = \frac{O_{i\cdot} \cdot O_{\cdot k}}{O_{\cdot\cdot}}$$

Thus, for example, from the first rows of the AB and AC marginals in Table 6, we get

$$E_{111} = \frac{(O_{11\cdot} = 65)(O_{\cdot 1} = 66)}{(O_{1\cdot\cdot} = 65 + 43 + 23 + 11)} = 30.2$$

$$E_{112} = \frac{65 \times 67}{162} = 30.7$$

and so on.

Note also that these estimates can be arrived at by applying the familiar procedure for calculating expected frequencies under the hypothesis of independence in contingency tables. Recall that the null hypothesis under discussion states that in each class of origin, the class of destination is probabilistically independent of family size. Under this null hypothesis, the conditional probability that a random observation in any given class of origin falls in any given class of destination is not dependent on the family-size category in which it falls. Once this is realized, it is seen that the expected frequency in each individual cell in Table 5 can be calculated by applying the familiar procedure for calculating expected frequencies under the assumption of row-column independence in contingency tables. All that we need to do is to apply this principle separately to the family-size-by-destination-class contingency tables under each class of origin. From the top quarter of Table 5, thus we get the expected frequency in the top-left-hand cell as  $65 \times 66 / 142$ , the corresponding figure in the cell right to it as  $65 \times 67 / 142$ , and so on.

The expected frequencies thus estimated are presented in Table 8. The chi-squared statistic calculated by the usual formula was found to be 121.9 with  $4 \times 3 \times 2 = 24$  d.f. The probability level of this chi-squared value is extremely small. This calls for the rejection of the null hypothesis that the ABC and BC interactions are together non-existent. The data are thus found to be consistent with the hypothesis that within each class of origin the conditional probability that a random observation falls in a given class of destination is

dependent on family size. I am inclined to regard this as a confirmation of the classical form of the "mobility-hypothesis" which holds that the chance of mobility is dependent on the family size. It is true that the existence of dependence between the chance of mobility and family size is statistically identical to observing that the destination status is associated with family size. What specific substantive interpretation this can be given depends upon the abstract experimental model the analyst is willing to impose on the data. It is in this connection that the discussion presented above becomes relevant. Under the SSDR (single-stimulus-double-response) model, the dependence between family size and destination status within each origin class will lead to the inference

- (1) that the chance of mobility is not independent of family size within each origin class (which is what the classical mobility-fertility hypothesis holds), *and*
- (2) that fertility is influenced by (the class-linked family-building habits of) the destination class.

Under Model (1), on the other hand, the same data would lead exclusively to the second of the above two inferences. So, from the point of view of substantive interpretation, the question boils down to which model, Model (1) (Fig. 1a) or the SSDR model (Fig. 1b), is considered to be more valid as the replacement of the concrete data set at hand. It is worth emphasizing that in making a choice between these models, the analyst would have to depend upon extra-statistical considerations. In the case of Berent's data, for example, as mentioned earlier, a relevant question to ask is whether mobility can be assumed to have taken place prior to family formation and/or the starting of family growth. If the answer is affirmative, Model (1) can be taken to be more appropriate than the SSDR model. If, on the other hand, there is reason to believe that mobility occurred as the family was growing, the SSDR model is to be preferred to Model (1).

### **Concluding Remarks**

The concern in this paper has been with abstract experimental models that can be employed as a valid replacement of mobility-fertility data of the type reported by Berent (1951). The models suggested by Berent (1951), Duncan (1966), and Hope (1971) were examined, and it was shown that as far as the underlying abstract experimental design is concerned, the authors are essentially in agreement. All of them implicitly assume an asymmetrical relationship between mobility and fertility, which amounts to stipulating that mobility occurs, if at all it does, prior to family formation and/or growth. I have taken the position that with reference to data sets such as Berent's (1951), it would be more valid to assume that mobility (class of destination) and fertility are symmetrically related responses (dependent variables) while class of origin is the stimulus (independent variable). It was shown that if this position were accepted as valid, the classical mobility-fertility hypothesis should be translated into the hypothesis that the ABC and BC interactions are simultaneously non-existent (where A=class of origin, B = class of destination, and C=family size) rather than as has been done by Duncan (1966) and Hope (1971) into the hypothesis that the ABC interaction is non-existent.

It should be emphasized that it was not my intention to criticize the work of Duncan (1966) or Hope (1971). My primary objective in this paper has been to draw attention to an

often neglected first step in data analysis, namely, the examination of the validity of the abstract experimental model with which the analyst replaces his real data before applying statistical tests. It would be appropriate to refer in this connection to Professor Barnard, who wrote more than a quarter of a century ago about the confusion that may arise if the analyst fails to examine the validity of the abstract experimental model he chooses to employ as the replacement of the concrete data set before him. He wrote:

"It is obvious enough that the first step in the statistical treatment of [a data set] may be represented as the replacement of the concrete [data by] an 'urn-experiment'; but the implications of this have not always had the continuous attention they deserve. Once the abstract picture has been formed the analysis of it is largely a matter of pure mathematics. What distinguishes the statistician from the pure mathematician, in this connection, should be the statistician's ability to form *valid* abstract pictures of concrete cases, and his clear recognition of the limits of validity of his abstract pictures. Yet we find relatively little discussion in statistical text-books of the process of formation of these abstract pictures."

Although Barnard was primarily concerned with the analysis of the 2 x 2 contingency table, the ideas expounded by him are relevant to the analysis of any concrete data set.

Before concluding there are a number of points that deserve to be emphasized. It has been remarked above that under the SDDR model (Figure 1b) the hypothesis that the ABC and BC interactions are simultaneously non-existent is a valid statistical translation of the essential idea, except for the causal implication, behind the classical mobility-fertility hypothesis. The test of this hypothesis, as expounded above, consists in investigating whether family size and destination class are associated within each of the origin classes. It will be instructive to compare the null hypothesis employed above to test this hypothesis with the null hypothesis that the P's are equal in Model (1). It is easily seen that the two null hypotheses are parallel. This raises the question whether an appropriate test of the null hypothesis that the ABC and BC interactions are simultaneously non-existent can be accomplished via the analysis of variance technique which employs Model (1). The answer is affirmative. If Model (1) is found to fit the data, then a test of the null hypothesis that the P's are equal would provide the required test.<sup>1</sup>

Another point that deserves to be mentioned here concerns the substantive meaning of the ABC interaction in Table 5 (or, parallel to it, of nonadditivity in Table 1.) We have already seen that statistically speaking if B (destination status) and C (family size) are more strongly associated in some classes of A (origin status) than in others, the ABC interaction is said to exist. To fix our ideas, let us consider the figures given in Table 9. Imagine that these figures represent the conditional probabilities that any random observation falls in a given destination class, within any given category of class of origin by family size. The figures show how these probabilities are associated (in this hypothetical case) with family size. It is easily seen that the association between these probabilities and family size is strongest among those born in Class 1 and weakest among those born Class 4. Hence we have here a data situation in which the ABC interaction is present. As the figures clearly indicate, the risk of down-mobility, if family size is large, is highest among those born in Class 1, slightly lower among those born in Class 2, and so on. The prospect of up-mobility,

on the other hand, is smallest, if family size is small, among those born in Class 4, slightly higher among those born in Class 3, and so on. An implication of such a pattern is that any given prospect of up-mobility is associated with much smaller family size among the lower classes than among the middle classes. The pattern also implies that the risk of down-mobility that those with large families may find themselves exposed to vary between classes of origin.

What has been said above is enough to demonstrate that the presence of ABC interaction in data sets such as the ones reported by Berent (1951) represent the presence of class differences in the cost, in terms of family size reduction, of achieving up-mobility or of preventing down-mobility. Whether this is what the classical mobility hypothesis implies is an open question. I have taken the position herein that this is not the implication of the classical form of the mobility-fertility hypothesis.

As a final remark, it is worth emphasizing that the SDR model fails to recognize the possibility that fertility and mobility may be related causally in a feedback fashion, fertility affecting mobility, and mobility, in turn, affecting the subsequent family growth. It is possible to modify the SDR model in such a manner as to take into account the feedback relationship between fertility and mobility. Any attempt to expound here how to do this would take us beyond the scope of this paper.

Figure 1a. Model 1

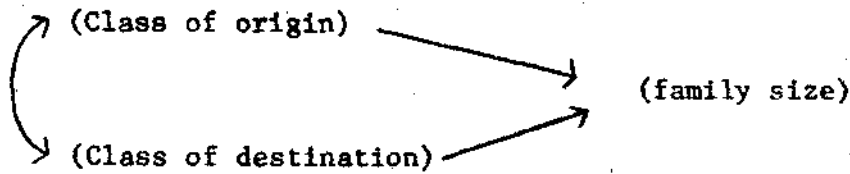
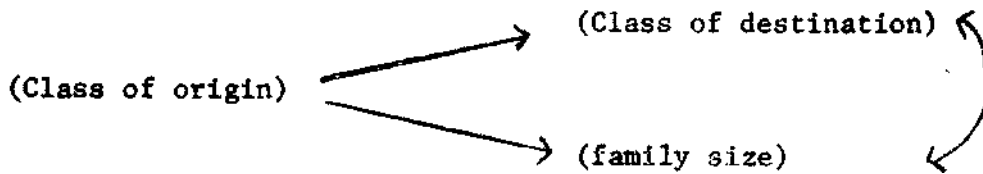


Figure 1b. Single-stimulus double response (SSDR) model.



Note: The causal connections implied by Model 1 are depicted in Fig. 1a. (The curved symmetrical arrow connecting class of destination reflects the association between them revealed in the unequal numbers of observation in the cells in Table 1.) Fig. 1b shows the causal connections implied by the single-stimulus double-response (SSDR) model. Both these models recognize that there is some association between class of origin and class of destination. Model 1 (Fig. 1a) treats this association as symmetrical, whereas SSDR model (Fig. 1b) treats it as asymmetrical. Both the models recognize that there is some association between class of destination and family size. Model 1 (Fig. 1a) treats this association as asymmetrical, whereas SSDR model (Fig. 1b) treats it as symmetrical. SSDR model (Fig. 1b) explicitly recognizes that family size may affect the chances of mobility while class-linked habits may influence fertility, whereas Model 1 (Fig. 1a) ignores the possibility that family size may affect the chances of mobility.

**FOOTNOTE**

1. The suggestion that a test based on Model 1 can be employed to shed light on the validity of a hypothesis which has its basis on the SSDR model may seem inconsistent with the emphasis laid throughout this paper on the need to distinguish Model 1 from the SSDR model. A little reflection would reveal, however, that there is nothing illogical about this suggestion. We know, for example, that the test of independence in the 2 x 2 table

a	b	(a+b)
c	d	(c+d)
(a+c)	(b+d)	(a+b+c+d)

is logically the same as the test of the hypothesis of no difference between the proportions  $a/(a+c)$  and  $b/(b+d)$ . But the former test is that of the symmetrical relation between the rows and columns whereas the latter test is that of an asymmetrical relationship. It is perhaps worth adding that many research workers who have investigated the mobility-fertility hypothesis have adopted procedures analogous to that of the testing of the hypothesis that P's in Model 1 are equal to one another (see, for example, Westoff, et al., 1963).

**Table 1—Average number of live births per couple and the number of couples in the sample in each origin-destination category.**

Origin (= husbands father's social, <i>i.e.</i> , occupational, class)	Destination (= husband's social, <i>i.e.</i> occupational class at the time of the survey)				
	1 (High)	2	3	4(Low)	All
(a) Average number of live births 1 (High)	1.74	1.79	1.96	2.00	1.81
2	2.05	2.14	2.51	2.97	2.38
3	1.87	2.01	2.67	3.69	2.81
4 (Low)	2.40	3.20	3.22	3.68	3.44
All	1.88	2.17	2.73	3.56	2.77
(b) Number of couples in the sample 1 (High)	65	43	23	11	142
2	38	197	150	68	453
3	37	154	431	244	866
4 (Low)	5	45	162	260	432
All	145	439	766	543	1,893

Source: Berent (1591), Table 1, p. 247.

**Table 2—Distribution of couples by class of origin, class of destination, and number of live births**

A Class of origin	B Class of destination	Number of live births											n	Mean
		0	1	2	3	4	5	6	7	8	9+			
1	1	10	17	23	13	1	0	0	1	0	0	65	1.74	
	2	5	18	10	7	1	1	0	0	0	1	43	1.79	
	3	4	7	6	3	1	0	1	1	0	0	23	1.96	
	4	2	3	3	2	0	0	0	1	0	0	11	2.00	
2	1	5	14	8	8	0	0	0	1	1	1	38	2.05	
	2	37	49	49	36	9	1	1	5	5	5	197	2.14	
	3	20	38	33	28	10	4	3	5	5	4	150	2.51	
	4	7	13	14	11	7	6	4	3	2	1	68	2.97	
3	1	5	15	8	7	0	0	0	0	1	1	37	1.87	
	2	22	43	49	26	5	0	0	3	4	2	154	2.01	
	3	58	92	94	77	34	23	15	15	15	8	431	2.67	
	4	19	29	35	45	32	27	22	17	9	9	244	3.69	
4	1	0	2	1	1	0	1	0	0	0	0	5	2.40	
	2	3	9	9	8	4	4	3	2	2	1	45	3.20	
	3	13	29	31	29	17	13	12	8	7	3	162	3.22	
	4	16	26	33	37	30	26	23	16	11	2	220	3.68	

Source : Constructed so as to be consistent with Table 1 and Table 4 of Berent (1951).

**Table 3—Distribution of marriages by direction of movement and family size**

Direction of Movement	Number of children			
	0-1	2-3	4 or more	All
Up	160	185	96	441
Static	305	362	246	913
Down	165	197	177	530
Total	630	744	519	1893

Source : Berent (1951) Table 4, p. 248.

**Table 4—Results of fitting model (1) to the data in table 2**

Source	Sum of squares	d.f.	Mean sum	F-ratio
Due to Model (1)	729.60	6	121.60	-
Due to non-additivity	66.72	9	7.41	1.61
Within cell variation Total	8660.38 9456.70	1877	4.61	-

**Table 5—Frequency distribution of couples by class of origin, class of destination, and family size**

A Class of Origin	B Class of Destination	C Family Size :			Total
		Small (0-1)	Medium (2-3)	Large shift (4 or more)	
1	1	27	36	2	65
	2	23	17	3	43
	3	11	9	3	23
	4	5	5	1	11
	Total	66	67	9	142
2	1	19	16	3	38
	2	86	85	26	197
	3	58	61	31	150
	4	20	25	23	60
	Total	183	187	83	453
3	1	20	15	2	37
	2	65	75	14	154
	3	150	171	110	431
	4	48	80	116	244
	Total	283	341	242	866
4	1	2	2	1	5
	2	12	17	16	45
	3	42	60	60	162
	4	42	70	108	220
	Total	98	149	185	1893

**Table 6—Marginals calculated from Table 5**

(i) AB Marginals

A	B			
	1	2	3	4
1	65	43	23	11
2,	38	197	150	60
3	37	154	431	244
4	5	45	162	220

(ii) AC Marginals

A	C		
	1: small	2: medium	3: large
1	66	67	9
2	183	187	83
3	283	341	242
4	98	149	115

(iii) BC Marginals

B	C		
	1: small	2: medium	3: large
1	68	69	8
2	186	194	59
3	261	301	204
4	115	204	248

**Table 7—Estimates of expected frequencies in individual cells of Table under the hypothesis that the ABC interaction is nonexistent**

A: Class of Origin	B: Class of Destination	C: Family Size		
		1 :small	2: medium	3: large
1	1	31.8	31.3	1.9
	2	20.4	20.2	2.4
	3	10.1	10.5	2.4
	4	3.7	5.1	2.2
2	1	18.1	17.5	2.4
	2	87.9	85.4	23.7
	3	58.5	59.5	32.0
	4	18.5	24.6	24.8
3	1	16.2	17.7	3.0
	2	62.4	68.5	23.2
	3	148.4	170.5	112.1
	4	56.0	84.3	103.7
4	1	1.9	2.5	0.6
	2	15.3	20.0	9.8
	3	44.1	60.5	57.4
	4	36.7	66.0	117.2

**Table 8—Estimates of expected frequencies in individual cells of Table 5 under the hypothesis that ABC and BC interactions are nonevistent**

A: Class of Origin	B: Class of Destination	C: Family Size	1: small	2: medium	3: large
1	1		30.2	30.7	4.1
	2		20.2	20.3	2.7
	3		10.7	10.9	1.5
	4		5.1	5.2	0.7
2	1		15.4	15.7	7.0
	2		79.6	81.3	36.1
	3		60.6	61.9	27.5
	4		22.5	28.1	12.5
3	1		12.1	14.6	10.3
	2		55.3	60.6	43.0
	3		140.8	169.7	120.4
	4		79.7	96.1	68.2
4	1		1.1	1.7	2.1
	2		10.2	15.5	19.3
	3		36.8	55.9	69.4
	4		49.9	75.9	94.2

Table 9—Hypothetical conditional probabilities of any random observation falling in a given class of destination, conditional on class of origin and family size.

A: Class of Origin	C: Family Size	B: Class of destination				Total
		1	2	3	4	
1	Small	0.8	0.2	0.0	0.0	1.0
	Large	0.2	0.8	0.0	0.0	1.0
2	Small	0.2	0.5	0.2	0.1	1.0
	Large	0.0	0.3	0.6	0.1	1.0
3	Small	0.0	0.1	0.5	0.4	1.0
	Large	0.0	0.0	0.3	0.7	1.0
4	Small	0.0	0.0	0.05	0.95	1.0
	Large	0.0	0.0	0.0	1.0	4.0